विद्या सर्वार्थि साधिका

ANANDALAYA
PERIODIC TEST - 2
Class: XII

Subject : Mathematics (041) M.M : 80
Date : 03–10–2024 Time : 3 Hours

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.
7. All Questions are compulsory. However, an internal choice in 2 questions of 2 marks, 2 questions of 3 marks and 2 Questions of 5 marks has been provided.

SECTION – A

1. For what value of x, the given matrix $\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix? (1)

(A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$

2. If the matrix $A = \begin{bmatrix} 0 & a & 8 \\ 5 & 0 & 12 \\ b & -12 & c \end{bmatrix}$ is a skew symmetric matrix then $a + b + c =$ _____. (1)

(A) 13 (B) -13 (C) -3 (D) 3

3. Write the principal value of $\tan^{-1}\left[sin\left(\frac{-\pi}{2}\right)\right]$. (1)

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) 1

4. The function $f(x) = |x-1| + |x+1|$ $for\ all\ x \in R$ is _____. (1)

(A) continuous , but not differentiable at $x = 1\ and\ x = -1$
(B) differentiable , but not continuous at $x = 1\ and\ x = -1$
(C) continuous , but not differentiable at $x = 1$ only
(D) neither continuous nor differentiable at $x = 1\ and\ x = -1$

5. If $y = cos(sinx^2)$, find $\frac{dy}{dx}$ $at\ x = \sqrt{\frac{\pi}{2}}$. (1)

(A) 1 (B) -1 (C) 0 (D) $-\frac{1}{2}$

6. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the *x*-coordinate. (1)

(A) $(4, 11)\ and\ \left(-4, \frac{31}{3}\right)$ (B) $(4, -11)\ and\ (-4,\ 4)$
(C) $(4, 11)\ and\ \left(-4, -\frac{31}{3}\right)$ (D) $(4, -11)\ and\ \left(-4, -\frac{31}{3}\right)$

7. The function $f(x) = x^2 - 8x$ is strictly decreasing on _____. (1)

(A) $(4, \infty)$ (B) $(-\infty,\ 4)$ (C) $(8, \infty)$ (D) $(-\infty, -4)$

8. $\int \text{cotx log(sinx)}\ dx =$ _____ (1)

(A) $\frac{1}{2}[\log(cosx)]^2$ (B) $\frac{1}{4}[\log(sinx)]^2$ (C) $\frac{1}{2}[\log(cotx)]^2$ (D) $\frac{1}{2}[\log(sinx)]^2$

9. Find $\frac{dy}{dx}$, if $xy = 100(x+y)$ (1)

(A) $\frac{(100-y)}{(100-x)}$ (B) $\frac{(100-y)}{(x-100)}$ (C) $\frac{50}{(x-100)}$ (D) $\frac{(y-100)}{(x-100)}$

10. If the matrix $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$, is the square root of the 2×2 identity matrix, then find the relation between a, b and c. (1)

(A) $a^2 + bc = 0$ (B) $a^2 + bc = 1$ (C) $a^2 - bc = 0$ (D) $b^2 + ac = 1$

11. Let $X = \{-1, 0, 1\}$ and $Y = \{0, 1, 2\}$ a function $f: X \to Y$ defined by $f(x) = x^2$ is _____ (1)

(A) one-one and onto (B) one-one but not onto
(C) many one – onto (D) many one – into

12. Evaluate : $\int \frac{tan^5 x\, sec^2 x}{tan^{12}\, x + 1} dx$ (1)

(A) $\frac{1}{6}\tan^{-1}(tan^6 x) + C$ (B) $\tan^{-1}(tan^6 x) + C$
(C) $\frac{1}{6}\log(tan^6 x) + C$ (D) $\frac{1}{6}\tan^{-1}(tan^{12} x) + C$

13. Given that $\tan^{-1}(tan\theta) = \theta$, then $\theta \in$ _____. (1)

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (C) $\left[-\frac{\pi}{2}, 0\right]$ (D) $(0, \pi)$

14. If $f(x) = x\tan^{-1} x$ then $f'(1) =$ _____. (1)

A) $1 + \frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2} + \frac{\pi}{4}$

15. Find the value of k so that $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2} & , \ x \neq 2 \\ k & , \ x = 2 \end{cases}$ is continuous at 2. (1)

(A) -7 (B) 7 (C) 2 (D) 1

16. The value of $\int_0^{\frac{\pi^2}{4}} \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx =$ _____ (1)

(A) -2 (B) 1 (C) 2 (D) π

17. If $\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3}\sin^{-1}(ax) + C$, find the value of a is _____. (1)

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) $\frac{9}{4}$

18. Let N be the set of natural numbers and relation R on N be defined by $R = \{(x, y): x, y \in N, x + 4y = 10\}$. R is _____. (1)

(A) reflexive (B) symmetric
(C) not reflexive and not symmetric (D) reflexive but not symmetric

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

19 (A) : In set $A = \{a, e, i\}$ a relation R defined as R = $\{(i, i), (a, a)\}$ is reflexive. (1)
(R) : A relation R is reflexive in set A if $(a, a) \in R$ for all $a \in A$.

20 (A) : The points $A\ (2, 3),\ B\ (-1, -2)\ and\ C(5,\ 8)$ are collinear. (1)
(R) : The points $A\ (x_1,\ y_1),\ B\ (x_2,\ y_2)\ and\ C(x_3,\ y_3)$ are collinear if and only if area of $\Delta\ ABC$ is zero.

SECTION – B

21. Find the principal value of $\tan^{-1}\left\{2\ sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$. (2)

22. Evaluate : $\int \sqrt{1 + 2\ tanx\ (tanx + secx)}\ dx$. (2)

OR

Evaluate : $\int_0^{\frac{\pi}{4}} (tan^2x + tan^4x)\ dx$.

23. Show that the points $(a + 5,\ a - 4), (a - 2,\ a + 3)\ and\ (a,\ a)$ do not lie on a straight line for any value of a. (2)

OR

If A_{ij} is the co- factor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \times A_{32} + a_{23} \times A_{23}$.

24. A function $f: N \rightarrow N$ defined as $f(x) = 3x^2 - 7$. Show that $f(x)$ is one- one but not onto , For any two natural numbers x_1, x_2. (2)

25. Separate $\left(0, \frac{\pi}{2}\right)$ into subintervals in which the function $f(x) = sin3x$ is increasing or decreasing. (2)

SECTION – C

26. Check whether the relation *R* defined in the set {1, 2, 3, 4, 5, 6} as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive. (3)

27. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, such that $A^{-1} = kA$, then find the value of k. (3)

OR

If $A\ and\ B$ are two square matrices of order 2 such that $2A + 3B = I_2$ and $A + B = 2A^T$. Show that $A = \frac{1}{5} I_2$.

28. Evaluate : $\int_0^{\frac{\pi}{4}} \frac{tan^3x}{1 + cos2x}\ dx$ (3)

OR

Evaluate : $\int \frac{\sqrt{tanx}}{sin2x}\ dx$

29. If $A = \begin{bmatrix} x & -3 & 1 \\ 2 & y & 1 \\ 1 & 1 & z \end{bmatrix}$ and $xyz = 7$, $x + y - 6z = 11$, find $A.adjA$ (3)

30. If $x = ae^{\theta}(sin\theta - cos\theta)\ and\ y = ae^{\theta}(sin\theta + cos\theta)$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ (3)

31. Find the intervals in which the function f given by $f(x) = 8 + 36x + 3x^2 - 2x^3$ is increasing or decreasing. (3)

SECTION – D

32. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations: (5)
$2x + y - 3z = 13$; $3x + 2y + z = 4$; $x + 2y - z = 8$

33. If $y = x\log\left(\frac{x}{a+bx}\right)$, *prove that* $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$. (5)

OR

If $x = a(\theta + sin\theta)$, $y = a(1 + cos\theta)$, find $\frac{d^2y}{dx^2}$ $at\ \theta = \frac{\pi}{2}$.

34. Evaluate : $\int \frac{x}{x^3+x^2+x+1}\, dx$. (5)

OR

Show that $\int_0^{\frac{\pi}{4}} \log(1 + tan\theta)\, d\theta = \frac{\pi}{8}\, log2$

35. If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, Find $A^2 - 5A + 4I$.Hence find a matrix X such that $A^2 - 5A + 4I + X = 0$. (5)

SECTION –E

36. Saurabh and Raji were playing Ludo at home. While rolling the die, Saurabh's sister Aditi observed and noted that a possible outcome of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let *A* be the set of players while *B* be the set of all possible outcomes.

$A = \{S, R\}$, $B = \{1, 2, 3, 4, 5, 6\}$

i) Let $R : B \to B$ be defined by $R = \{(x, y) : y\ is\ divisible\ by\ x\}$. Is the relation *R* symmetric? (1)

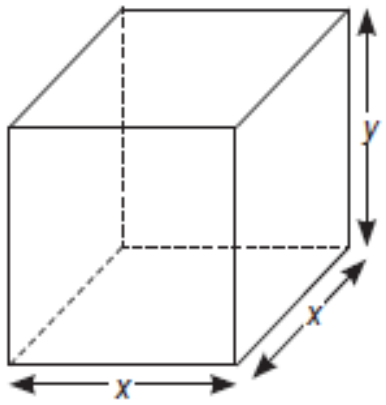
ii) Write the identity relation from $B \to B$ (1)

iii) Let *R* be a relation on *B* defined by *R* = {(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)}. Then check whether *R* is an equivalence relation. (2)

OR

iii) Aditi wants to know the number of relations from *A* to *B*. How many number of relations are possible?

37. A given quantity of metal sheet is to be cast in to an open tank with a square base and vertical sides as shown. Where x be the side of base square and y be the vertical side. Answer the following based on above information.



i) If V represent the volume of tank, then find relation between $V, x\ and\ y$. (1)

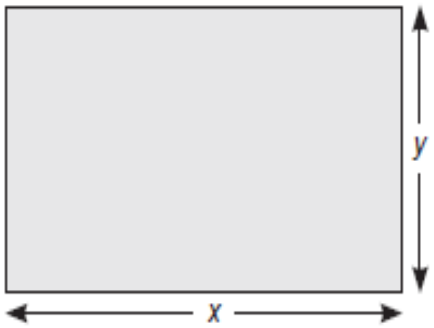
ii) Find the total area of tank, expressed as function of x, in terms of V . (1)

iii) Find x in terms of V when total surface area of tank is minimum. (2)

OR

iii) Represent the relation in $y\ and\ x$, so that the area is minimum.

38. Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain the same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.



Based on the information given above, answer the following questions

i) The equations in terms of x and y are _____ (2)

ii) Find the matrix equation represented by the given information. (2)